

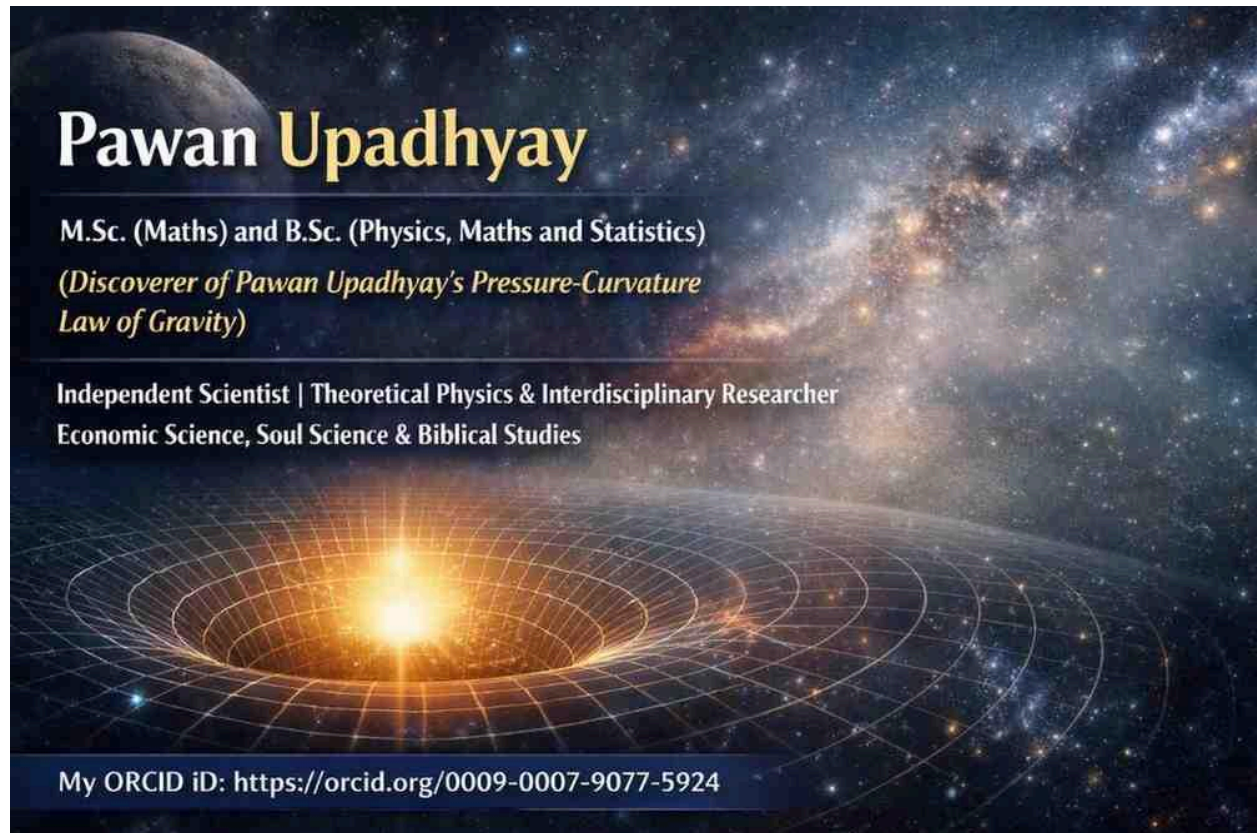
Explanation of the Formulas

$$F = f / V \text{ and } F = \rho a$$

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Force Density Definition

The field force density is defined as:

$$F = f / V$$

where

f = total force

V = volume

F = force per unit volume (force density)

This expression converts a total force acting on a body into a distributed field quantity.

Unit Check

Force (f):

Unit = Newton (N)

$$1 \text{ N} = \text{kg} \cdot \text{m} \cdot \text{s}^{-2}$$

Volume (V):

$$\text{Unit} = \text{m}^3$$

Therefore,

$$F = \text{N} / \text{m}^3$$

Substitute Newton:

$$F = (\text{kg} \cdot \text{m} \cdot \text{s}^{-2}) / \text{m}^3$$

$$F = \text{kg} \cdot \text{m}^{-2} \cdot \text{s}^{-2}$$

Thus, the unit of force density is:

$$\text{N/m}^3$$

or

$$\text{kg} \cdot \text{m}^{-2} \cdot \text{s}^{-2}$$

Dimensional Check

Force dimension:

$$[f] = \text{M L T}^{-2}$$

Volume dimension:

$$[V] = \text{L}^3$$

So,

$$[F] = (\text{M L T}^{-2}) / \text{L}^3$$

$$[F] = \text{M L}^{-2} \text{T}^{-2}$$

This is the correct dimension for force density.

Force Density from Mass Density

Starting from Newton's second law:

$$f = m a$$

Divide both sides by volume V:

$$f / V = (m / V) a$$

But mass density is defined as:

$$\rho = m / V$$

Therefore,

$$F = \rho a$$

Unit Check

Mass density (ρ):

$$\text{Unit} = \text{kg/m}^3$$

Acceleration (a):

Unit = m/s^2

Multiply:

$(\text{kg/m}^3) \times (\text{m/s}^2)$

$= \text{kg} \cdot \text{m}^{-2} \cdot \text{s}^{-2}$

$= \text{N/m}^3$

This matches the unit of force density.

Dimensional Check

Mass density dimension:

$[\rho] = \text{M L}^{-3}$

Acceleration dimension:

$[a] = \text{L T}^{-2}$

Multiply:

$(\text{M L}^{-3})(\text{L T}^{-2})$

$= \text{M L}^{-2} \text{T}^{-2}$

This exactly matches the dimension of force density.

Final Conclusion

$F = f / V = \rho a$

Unit: N/m^3

Dimension: $\text{M L}^{-2} \text{T}^{-2}$

Both expressions are dimensionally consistent and physically correct when ρ represents mass density.